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OBSERVABLE GRAVITATIONAL AND ELECTROMAGNETIC ORBITS AND TRAJECTORIES IN DISCRETE PHYSICS*

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ABSTRACT

Our discrete and finite version of relativistic quantum mechanics provides an elementary particle physics consistent with the standard model of quarks and leptons. Our recent relativistic calculation of the bound state spectrum of hydrogen has allowed us to make a combinatorial correction to the first order estimate of $1/\alpha = \hbar c/e^2 = 137$ derived from the *combinatorial hierarchy* and achieve agreement with experiment up to terms of order α^3. The same theory requires that to first order $\hbar c/Gm_p^2 = 2^{127} + 136 \simeq 1.7 \times 10^{38}$. Using the emission *and* absorption of spin 1 photons and spin 2 gravitons in this framework, we try to show that we can meet the three traditional tests of of *general relativity* — solar red shift, solar bending of light, and precession of the perihelion of Mercury. We predict that a *macroscopic* electromagnetic orbit would have four times the Sommerfeld precession for basically the same reason that Mercury has six times the Sommerfeld precession.

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1. INTRODUCTION

Connecting quantum mechanics with relativity theory has turned out to be difficult. We might even call it the "presenting problem" of 20^{th} Century physics. A way to meet the problem is to use a finite and discrete constructive theory[1-3] based on McGoveran's *ordering operator calculus* and the *combinatorial hierarchy* of Amson, Bastin, Kilmister and Parker-Rhodes. Following this program has led to a successful deduction of the Sommerfeld formula for the fine structure of hydrogen and a correlated *calculation* (!!) of the fine structure constant that is in agreement with experiment to seven significant (decimal) figures. We concluded that it might be worth while to try to calculate the solar red shift, the deflection of light by the sun, and the precession of the perihelion position of the orbit of Mercury using the same theory. This line of thought led to a new prediction of an effect that might be observed in *macroscopic* orbits of two isolated electrostatically charged systems.

The combinatory hierarchy $(3, 10, 137, 2^{127} + 136)$ already suggests that the scale constants of physics might be the result of asking how many cases can occur when we count (measure?) without imposing prior restrictions. The suggestive numerical "coincidences" $\hbar c/e^2 \simeq 137$ and $1.7 \times 10^{38} \simeq \hbar c/Gm_p^2 \simeq 2^{127} + 136$ and the standard connection $\hbar c/Gm_p^2 = (M_{Planck}/m_p)^2$ indicate that the theory we are talking about, if it works, unifies electromagnetism with gravitation. Since the theory as so far developed starts from standard "mass-length-time" physics, it has no place for more than three "dimensional" scales. Because it is a *fundamental* theory, once three unique identifications of how these scales enter the theory have been made all numbers in the theory are dimensionless ratios and must be *calculated* rather than taken from experiment. Hence, having identified the limiting velocity and the constant of action at very early stages in the construction, there is no place for a difference between "gravitational" and "inertial" mass; we do not have to postulate the "equivalence principle". Our theory is "born unified" both with respect to electromagnetism and gravitation *and* with respect to gravitation and inertia.

Given our unified theory the three classical tests of general relativity (solar red shift, solar deflection of light, Mercury perihelion shift) can be arrived at by simple heuristic arguments, which we present in the concluding section. One just complaint about these heuristic arguments is that we have mixed apples and oranges (and other more dubious fruits garnered from the tree of knowledge) in such a way as to lead to fruit salad. This sort of thing went on for a long time while quantum mechanics was being developed. One early practitioner of quantum mechanics in Copenhagen, who kept an iron horse shoe nailed over his (kitchen) door "to keep out the evil spirits" justified his practice by the remark "You don't have to believe in it for it to work." Perhaps we are still at that stage in the new theory. We hope in what follows to change your mind about our own practice. We try to do so by presenting a reasonably systematic development of the portions of our theory which bear most directly on the the specific problems adressed in this paper before presentingour way of looking at the three traditional tests of general relativity.

2. METHODOLOGY

The title of this conference — "Physical Interpretations of Relativity Theory"— was interpreted in various ways by the participants. Before presenting our calculations, we explain our own methodology. For us the *practice*[4] of physics when using McGoveran's modeling methodology[5] starts with a rough agreement as to what we are trying to model and how we can come to agree about success or failure in this enterprise. Such agreement is always hard to achieve without invoking arbitrary exclusions, or preferably engaging in several recursive passes through the steps mentioned below. A succeeding step is to formulate an uninterpreted but self consistent *representational framework* (R-frame) — for us this is McGoveran's *ordering operator calculus* [5]; Gefwert[4] would call formulating a representational framework the practice of *syntax.* These steps are incomplete for any problem, or clutch of problems, until one provides *rules of correspondence* between the practice and the representational framework— Gefwert would call this

practicing *semantics*— which satisfy the practitioners or lead them to abandon or modify the enterprise. For physics these rules of correspondence must obviously include the specification of relevant laboratory experiments referred to commonly understood and accepted dimensional standards and practical laboratory procedures (meter, kilogram, second, or any *three* dimensionally independent standards uniquely related to them) *and* agreement with experiment up to the ambiguities currently considered acceptable.

We part company with the thrust of the conference in two fundamental ways. We see the presenting problem of twentieth century physics as the reconciliation between quantum mechanics (discrete, global yet indivisible events measured by the quantum of action) and the implications of the limiting velocity for *causal* communication from which most treatments of relativity theory start. The second way in which we go outside most contemporary theoretical physics and mathematics, and not just outside the presuppositions of many participants here, is that we abandon the continuum from the outset by insisting that our theory be finite, discrete, finitely computable, contain absolute non-uniqueness, and be strictly constructive. In fact we see the source of many of the "paradoxes" of contemporary physics (and mathematics) as arising from trying to imbed quantum events (finite algorithms) in a structure which is taken *a priori* to be continuous; there is no physical procedure which can measure an "infinitesimal" or an "infinity" (no finite construction of the continuum).

3. LIMITING VELOCITY

Paradoxically, by abandoning the continuum we find a richer structure than that available in the valid portions of continuum theories. Take the question of the "limiting velocity", which has to be inserted in current theories as a brute fact or "just so story" rather than derived. Quoting from Ref. 3:

"In any finite and discrete theory such as ours any question as to whether a finite ensemble has a specific attribute can be answered "NO" or "YES". Thus,

with respect to any particular attribute and a well defined (strictly constructive) computational procedure, we can define an *attribute distance* relative to some reference ensemble by the number of computational steps it takes to bring the ensemble into local isomorphism with the reference ensemble. If we call the number of steps which increase the distance I and the number of steps which decrease the distance D, the attribute distance is $I - D$. If we take as our unit of time the computational step, this gives us the *attribute velocity* as $\frac{I-D}{I+D}$, which is obviously bounded by ± 1. [Note that] $I + D$ has to be greater than zero in a discrete and finite theory. Thus any attribute and any computational procedure specify a limiting velocity.

"If we use such a theory to model physics, we must specify which attributes in the theory are to correspond to those physical attributes which specify a physical object. In general these will specify different limiting velocities. Clearly the transmission of *causal* (i.e. physically effective) information between two physical objects will be limited by the minimum of these maximum velocities and can be identified with c, the physical limiting velocity...." [simply because the specification of a physical object requires the specification of *more* attributes than any partial list contains]

4. 3+1 SPACE-TIME

The casual way in which we have introduced *dichotomous choice* tends to conceal the implications. In appropriate *context* any sequence of questions raised in the theory can be mapped onto ordered strings of two symbols (eg *bit strings* containing only 0's and 1's). In the absence of further information, each string represents a sequence of Bernoulli trials with 0 and 1 representing the two possibilities. This has an extremely important consequence, which we call *McGoveran's Theorem*:

"The upper bound on the global d-dimensionality of a d-space of cardinality N with a discrete, finite and homogeneous distance function is 3 for sufficiently large N." (Ref. 5, Theorem 13, pp 59-60).

The argument can be sketched as follows. As has been noted by Feller,[6] if we have D *independent* sequences of Bernoulli trials, the probability that after n trials we will have accumulated the same number (k) of 1's is $p_D(n) = (\frac{1}{2^{nD}})\Sigma_{k=0}^{n}(\binom{n}{k})^D$. He then shows that the probability that this situation will repeat N times is strictly bounded by

$$P_D(N) = \Sigma_{n=1}^{N} p_D(n) < [\frac{2}{\pi D}]^{-\frac{1}{2}}\Sigma_{n=1}^{N} n^{\frac{1}{2}(D-1)}$$

Consequently for $D = 2, 3$, where $p_D(n) < n^{-\frac{1}{2}}, n^{-1}$, such repetitions can keep on occurring with finite probability, but for four or more independent sequences, this probability is strictly bounded by zero in the sense of the law of large numbers.

McGoveran[5] uses finite attributes, which can always be mapped onto ordered strings of zeros and ones, as the starting point for his ordering operator calculus. In order to introduce the concept of *dimensionality* into this space, he notes that we need some metric criterion that does not in any way distinguish one dimension from another. (In a continuum theory, we would call this the property of "homogeneity and isotropy"; we need it in our theory for the same reason Einstein did in his development of special relativity.) McGoveran discovered that by interpreting the coincidences $n = 1, 2, ..., N$ in Feller's construction as "metric marks" the metric space so constructed has precisely the discrete property corresponding to "homogeneity and isotropy" as just defined. Consequently Feller's result shows that in *any* finite and discrete theory, the number of independent "homogeneous and isotropic" dimensions is bounded by three! If we start from a larger number of independent dimensions using *any* discrete and finite generating process for the attribute ensembles, we find that the metric will, for large numbers, continue to apply to only three of them, and that what may have looked like another dimension is not; the probability of generating the next "metric" mark in any of the others (let alone all of them) is strictly bounded by $1/N_{MAX}$!

Of course the argument depends on the theory containing a *universal ordering operator* which is isomorphic to the ordinal integers. Further, since we know empirically that "elementary particles" are *chiral*, we will need three rather than

two "spacial" dimensions. Thus *any* discrete and finite theory such as ours when applied to physics must be globally described by three dimensions and a monotonically increasing order parameter. Consequently we are justified in constructing a "rule of correspondence" for our theory which connects the large number properties of our R-frame to *laboratory* (E-frame) $3+1$ space-time.

5. THE COMBINATORIAL HEIRARCHY

In the ordering operator calculus[5] the norm or resultant of two independent ("orthogonal") quantities can only be calculated when all three numbers satisfy the restrictions $(a-b)^2 \leq c^2 \leq (a+b)^2$, cyclic on a, b, c. The specific model used here is based on labeled bit strings $(\ell)_L \| (a)_n$ where $(x)_y = (..., b_i^x, ...)_y$; $b_i^x \in 0, 1, i = 1, 2, ..., y$. The first part of the string is called the *label* and the second part the *content*. Strings combine under *discrimination* (exclusive or, $\oplus$,...) according to $(xw)_n = (..., b_i^x + _2 b_i^w, ...)_n$ and under string concatenation according to $(x)_y \| (w)_z = (x, w)_{y+z}$ where $b_j^{x,w} = b_i^x, i, j = 1, 2, .., y; = b_i^w, i = 1, 2, .., z, j = i + y$. We define the *null string* $(0)_n$ by $b_i^0 = 0, i \in 1, 2, ..., n$ and the *anti-null string* $(1)_n$ by $b_i^1 = 1, i \in 1, 2, ..., n$. Since the operation $\oplus$ is only defined for strings of the same length we can usually omit the subscript n without ambiguity. The definition of discrimination implies that

$$(aa) = (0);\ (ab) = (ba);\ ((ab)c) = (a(bc)) \equiv (abc)$$

and so on.

The importance of closure under this operation was recognized by John Amson. It rests on the obvious fact that $(a(ab)) = (b)$ and so on. We say that any finite and denumerable collection of strings, where all strings in the collection have a distinct tag $i, j, k...$, are *discriminately independent* iff

$$(i) \neq (0) :\ (ij) \neq (0),\ (ijk) \neq (0), ...(ijk...) \neq (0)$$

We define a *discriminately closed subset* of non-null strings $\{(a), (b), ...\}$ as the

set with a single non-null string as member or by the requirement that any two different strings in the subset give another member of the subset on discrimination. Then two discriminately independent strings generate three discriminately closed subsets, namely

$$\{(a)\},\ \{(b)\},\ \{(a),(b),(ab)\}$$

Three discriminately independent strings give seven discriminately closed subsets, namely

$$\{(a)\},\ \{(b)\},\ \{(c)\}$$

$$\{(a),(b),(ab)\},\ \{(b),(c),(bc)\},\ \{(c),(a),(ca)\}$$

$$\{(a),(b),(c),(ab),(bc),(ca),(abc)\}$$

In fact x discriminately independent strings generate $2^x - 1$ discriminately closed subsets because this is simply the number of ways one can take x distinct things one, two, three,...,x at a time. Hence we can generate the sequence, known as the *combinatorial hierarchy*[7], $(2 \Rightarrow 2^2 - 1 = 3), (3 \Rightarrow 2^3 - 1 = 7)$ $[3 + 7 = 10], (7 \Rightarrow 2^7 - 1 = 127)$ $[10 + 127 = 137], (127 \Rightarrow 2^{127} - 1 \simeq 1.7 \times 10^{38})$ — $[3, 10, 137, 2^{127} + 136 \simeq 1.7 \times 10^{38}]$ in suspiciously accurate agreement with the "scale constants" of physics. The discovery of the combinatorial hierarchy was made by Parker-Rhodes in 1961[8]. The real problem is to find some "stop rule" that terminates the construction.

The original stop rule was due to Parker-Rhodes. He saw that if the discriminately closed subsets at one level, treated as sets of vectors, could be mapped by non-singular (so as not to map onto zero) square matrices having uniquely those vectors as eigenvectors, and if these mapping matrices were themselves discriminately independent, they could be rearranged as vectors and used as a basis for the next level. In this way the first sequence is mapped by the second sequence $(2 \Rightarrow 2^2 = 4)$, $(4 \Rightarrow 4^2 = 16)$, $(16 \Rightarrow 16^2 = 256)$, $(256 \Rightarrow 256^2)$. The

process terminates because there are only $256^2 = 65,536 = 6.5536 \times 10^4$ d.i. matrices available to map the fourth level, which are many too few to map the $2^{127} - 1 = 1.7016... \times 10^{38}$ DCsS's of that level. By now there are many ways to achieve and look at this construction and its termination.[9-13]

6. PROGRAM UNIVERSE

In order to generate a universe of strings which grows, sequentially, in either number (SU) or length (N_U) Mike Manthey and HPN created *program universe*[14]. The program is initiated by the arbitrary choice of two distinct bits, which become the first two strings in the universe. Entering the main routine at $PICK$, we choose two strings (i) and (j) and discriminate them. Whenever the two strings picked are identical, $(ij) = (0)_{N_U}$ and we go to TICK. TICK concatenates a single bit, arbitrarily chosen for each string, to the growing end, notes the increase in string length, and the program returns to $PICK$. The alternative route, which occurs when discrimination generates a non-null string, simply ADJOINs the newly created string to the universe, notes the increase in SU, and the program returns to $PICK$.

The method Manthey and HPN used to "construct" the hierarchy was to demonstrate explicitly that any run of PROGRAM UNIVERSE contained all we needed to extract some representation of the hierarchy and the label content scheme from the computer memory *without* affecting the running of the program. The obvious intervention point exists where a new string is generated, i.e. at ADJOIN. The subtlety here is that if we assign the tag i to the string $U[i]$ as a *pointer* to the spot in memory where that string is stored, this pointer can be left unaltered from then on. It is of course simply the integer value of $SU + 1$ at the "time" in the simulation [sequential step in the execution of that run of the program] when that memory slot was first needed. Of course we must take care in setting up the memory that *all* memory slots are of length $N_{max} > N_U$, i.e. can accommodate the longest string we can encounter during the (necessarily finite) time our budget

will allow us to run the program. Then, each time the program TICKs, , the bits which were present at that point in the sequential execution of the program when the slot $[i]$ was first assigned will remain unaltered; only the growing head of the string will change. Thus if the strings $i, j, k....$ tagged by these slots are discriminately independent at the time when the latest one is assigned, they will remain discriminately independent from then on.

Once this is understood the coding Manthey and HPN gave for our labeling routine should be easy to follow. We take the first two discriminately independent strings and call these the basis vectors for *level 1*. The next vector which is discriminately independent of these two starts the basis array for *level 2*, which closes when we have 3 basis vectors discriminately independent of each other and of the basis for level 1, and so on until we have found exactly $2+3+7+127$ discriminately independent strings. The string length when this happens is then the *label length* L; it remains fixed from then on. During this part of the construction we may have encountered strings which were *not* discriminately independent of the others, which up to now we could safely ignore. Now we make one *mammoth* search through the memory and assign each of these strings to one of the four levels of the hierarchy; it is easy to see that this assignment (if made sequentially passing through level 1 to level 4) has to be unique.

From now on when the program generates a new string, we look at the first L bits and see if they correspond to any label already in memory. If so we assign the content string to the *content ensemble* carrying that label. If the new string also has a new label, we simply find (by upward sequential search as before) what level of the hierarchy it belongs to and start a new labeled content ensemble. Because of discriminate closure, the program must eventually generate $2^{127} + 136$ distinct labels, which can be organized into the four levels of the hierarchy. Once this happens, the label set cannot change, and the parameters i for these labels will retain an *invariant* significance no matter how long the program continues to TICK. It is this invariance which will later provide us with the formal justification for assigning an invariant mass parameter to each string. We emphasize once

more that *what* specific representation of the hierarchy we generate in this way is irrelevant; any "run" of PROGRAM UNIVERSE will be good enough for us.

To summarize, the *label* strings $(\ell)_L$ of length L bits close under discrimination to form some representation of the four levels of the *combinatorial hierarchy* with $3, 7, 127$ and $2^{127} - 1$ elements, which terminates because of the mapping onto the sequence $4, 16, 256, 256^2$. This implies that $139 \leq L \leq 256$. In the absence of further information, the content string will be any one of the 2^n possible strings of length n.

7. LORENTZ TRANSFORMATIONS

Physical interpretation of the labels, based on the demonstration of *conservation laws* in bit string events[2], identifies level 1 with chiral neutrinos, level 2 with electrons, positrons and gamma rays, and level 3 with two quarks in a color octet and their associated gluons. This pattern repeats at level 4, generating up to 16 generations of quarks and leptons which couple to low energy phenomena with rapidly and monotonically decreasing probability. The identification of the coulomb interaction as occurring with probability 1/137 (because one label can represent only one of the $2^2 - 1 + 2^3 - 1 + 2^7 - 1 = 137$ labels generated when level 3 closes) for charged leptons and the appropriate 1/3 or 2/3 factor for quarks is preserved across generations. Since we are concerned in what follows with hydrogen, the unification of the coulomb interaction between electron and proton (i.e. a *uud* bound state) in the first quark-lepton generation is all we will need beyond the basic combinatorial structure of the labels. We can anticipate corrections due to weak-electromagnetic unification to occur with probability of order $m_p^2\sqrt{2}G_F|sin^2\theta_{Weak} - \frac{1}{4}|$, which is beyond the accuracy of the calculation made here.

The physical interpretation of the content strings was originally made in terms of a "random walk" between "events" using the *ordinal* integral positions of the bits along the string to specify invariant *sequence*. Counting the number of 1's in

the string as k (and hence the number of 0's as $n-k$), and taking as reference string any string with $2k = n$, the *attribute distance* of any string, defined as the number of steps away from minus the number of steps toward the reference string is $2k - n$, consistent with our general discussion given above. Introducing physical (i.e., mass, length, time) dimensions by an invariant length λ, this distance $x/\lambda = 2k - n$, since each step is executed at the limiting velocity c, $ct/\lambda = n$.

We define *events* by the restriction that $(abc) = (0)$, or $(abcd) = (0)$. We consider two sequential events separated by n steps connecting two strings with the same label. Take as the origin $(x_0, t_0) = (0, 0)$ and call the event of interest to us (x, t). Then the average velocity between the two events was $\beta c = (\frac{2k}{n} - 1)c$. The invariant interval connecting the events is $s^2/\lambda^2 = (c^2t^2 - x^2)/\lambda^2 = 4k(n-k) = (1-\beta^2)n^2$. Going to light cone coordinates, we can make a distinction between two factors of the invariant interval by defining $d_+ = 2k = (1+\beta)n, d_- = 2(n-k) = (1-\beta)n$; we see that the transformation $k' = \rho k, n' - k' = \rho^{-1}(n-k)$ leaves s^2 invariant. Since $d_+/d_- = (1+\beta)/(1-\beta)$, the transformation to the rest system defined by $d'_+/d'_- = 1$ is given by $\rho^2 = (1-\beta)/(1+\beta)$, from which the Lorentz transformations for x and ct follow immediately.

The invariance of the label string under our generation procedure, and the conservation laws for quantum numbers which participate in events, allows us to define a dimensional invariant for each label, more specific than the global limiting velocity c. In the discussion above, we used a space-time language, and referred to an invariant length λ. But, in current particle physics, velocity (in units of the limiting velocity) has the same value in energy-momentum space as in space-time. Hence, we could just as well have taken our invariant connected to the label as the invariant mass-energy mc^2 rather than λ. Then we can understand quantization either as taking the invariant length $\lambda = h/mc$ or as $E = h\nu$ with the invariant frequency $\nu = mc^2/h$; this is a distinction without a difference. Since each step is executed at the limiting velocity, a labeled history which starts at rest $[2k = n]$ acquires a momentum mc by taking the first step, and must take a second step to return to rest. Hence it makes a circuit which encloses an area h in phase space; for

longer circuits this will be nh. We have extended Bohr-Sommerfeld quantization to a relativistic free particle. At each step the value of position and velocity depend on the sequence and the sense of the circuit; this insures that position and momentum do not commute. It is this fact which allows us to ascribe a common origin to relativity and quantum mechanics in any finite, discrete, constructive theory.

The fact that we have Bohr-Sommerfeld quantization in a relativistic quantum theory encouraged us to tackle the fine structure problem — because this was all that was available to Sommerfeld! Since his result is valid up to corrections of order α^3 (eg the Lamb shift) using the accepted non-integral value for $1/\alpha$, this meant we couldn't just use the first otder value of 137. The beauty of DMcG's approach[15] is that it solves both problems at once.

8. FINE STRUCTURE OF HYDROGEN

We have seen that for a particle at rest ($2k = n$), there is a *zitterbewegung* with momentum $\pm mc$ which encloses an area of some integral multiple of h in phase space; hence Bohr-Sommerfeld quantization for a relativistic free particle. This periodicity is a special case of the periodicity for any finite (average) velocity $\beta = \frac{2k_0}{n_0} - 1$ since $k = Nk_0, n = Nk_0$ defines a periodicity N which leaves the velocity invariant. This defines the coherence length h/p, and allows events to occur only at positions an integral number of deBroglie wavelengths apart.

To extend this analysis to a coulomb bound state with system mass $\mu = \frac{mM}{m+M}$ we take from our interpretation of the combinatorial hierarchy the fact that only 1 in 137 of the events will be a coulomb event, the others averaging out in the first stage of the analysis; in other words

$$137N_B \ steps = 1 \ coulomb \ event$$

This means that we now have two frequencies (in dimensional units of $\mu c^2/h$), the *zitterbewegung* frequency corresponding to the rest mass, which we take to be unity,

and the coulomb frequency $1/137N_B$. Since these two motions are incoherent, the frequencies must be added in quadrature subject to the constraint on the energy E defining a bound state that in the rest system $E/\mu c^2 < 1$. Hence, $(E/\mu c^2)^2[1 + (1/137N_B)^2] = 1$. In the language of the ordering operator calculus, this is simply the normalization of the metric corresponding to the energy attribute under the appropriate constraint. If we take $e^2/\hbar c = 1/137$, this is just the relativistic Bohr formula[16].

In either the non-relativistic Bohr theory or the non-relativistic Schroedinger equation, the coulomb problem suffers from a degeneracy between the principle quantum number N_B and the orbital angular momentum quantum number ℓ, because the energy depends only on the principle quantum number, or, in the correspondence limit, on the semi-major axis of the ellipse. Thinking semi-classically, Bohr[16] and Sommerfeld saw that the relativistic mass increase, which is most important at perihelion in elliptical orbits, would break this degeneracy, and Sommerfeld[17] computed the effect. Dirac[18] arrived at the same formula in what appears to be a very different way, but one which also depends on lifting the degeneracy between two integers. For both Sommerfeld and Dirac the problem was, in a sense, easier than for us because in conventional theories irrational, transcendental, "empirical", ... numbers live in a different world than the finite integers. Their methodology allows these non-constructive entities to enter the argument at appropriate points. We must face a harder problem in our theory.

Let j be an integer, and let successive values of s differ by integers so that $s = n + s_0$. Although s_0 is rational, it lifts the degeneracy by being non-integral. If j and s_0 differed *only* by a rational fraction rescaling would restore the degeneracy. Hence the 137 coulomb rescaling from the combinatorial hierarchy, or any other single integral rescaling, is not enough to meet the problem posed. If we combine the two independent integer (except for s_0) counts by starting them off as close as we can while maintaining the distinction (i.e. "synchronize" the counting), we can require that s_0 be the value closest to j that s can have. This can happen in two distinct ways. There is no way in the problem posed that we can directly

observe the "synchronization" of the two periods, and both possibilities correspond to "coulomb events". We can either assume that the synchronization corresponds to $137j\frac{steps}{(coulomb\ event)} + 137s_0^+\frac{steps}{(coulomb\ event)} = 1 + \epsilon$ or to $137j\frac{steps}{(coulomb\ event)} - 137s_0^-\frac{steps}{(coulomb\ event)} = 1 - \epsilon$ where ϵ is some rational fraction less than unity.

Here we must use care because these two equations have different meanings and cannot simply be interpreted as if they represented numerical quantities which can be combined by linear operations. As we saw in our derivation of the relativistic Bohr formula, independent frequencies must be combined in quadrature, so we form the product defining the squares: $137^2j^2 - 137^2s_0^2 = 1 - \epsilon^2$. Note that the two factors of this equation are the conditions on j and s stated above. With j fixed, the value of s for which both hold is s_0. Since j is to be the norm to which we refer, we form $j^2 - s_0^2 = (1 - \epsilon^2)/137^2 = a^2$. Taking $s = n + s_0$ as the appropriate number to define internal frequency for the bound state, we can follow our discussion above for the single frequency case and conclude that

$$(E/\mu c^2)^2[1 + a^2/(n + \sqrt{j^2 - a^2})^2] = 1$$

This is precisely the Sommerfeld formula, provided we can interpret a^2 as α^2 (to order a^3 or α^3) and know how to take the square root in our discrete theory.

In order to understand how we can have two independent rational frequencies in our theory of this problem, we have to go back to where the 137 came from. In the absence of other information, the $3 + 7 + 127$ labels have to be generated for each of the two labeled strings which are coupled by the two coulomb events that (minimally) allow a bound state to be specified. But, if the end result is to be distinct, the way this is done the first time must be distinct from the way it happens the second time. For both events to be coulomb, the second time through the first two levels must have closed, so only 1 in 127 events would correspond to an indistinguishable repetition of the first process. Hence the population from which a coulomb bound state event is selected is reduced by 1 in 127 compared to statistical independence; this is standard statistics for sampling without replacement. But

for two spin 1/2 particles (electron and proton) only 1 in 16 possibilities out of the spin, particle-antiparticle, dichotomies will also coincide; the null case cannot occur in our scheme, leaving only 1 in 15×127 cases to be excluded. We conclude that the expectation of the "second" event being degenerate with the "first" event is just $1/(15 \times 127)$, which defines $2\epsilon = \frac{1}{15\times127}$ as the interval around unity by which $137^2 s_0^2$ can differ from $137^2 j^2$. In the physical situation however, we are only interested in the portion that occurs within the period for j, namely $1 - \epsilon$. Therefore the number of steps that are neither part of j or s_0 is $\epsilon = \frac{1}{2} \cdot \frac{1}{127\times15}$.

This two factor analysis of the way ϵ relates to the normalization equation $j^2 - s_0^2 = a^2$ raises another subtle point. When experimentalists use the Sommerfeld formula and the fine structure spectrum of hydrogen to evaluate α, they fit their results to α^2 and then take the square root. In order for this to correspond to the calculation we have made, we must take $a^2 = (1-\epsilon)^2/137^2$, and we expect them to find that $\frac{1}{\mathbf{a}} = \frac{137}{1-\frac{1}{30\times127}} = \mathbf{137.0359674}...$ in comparison to the accepted empirical value[19] 137.035963(15).

9. THE THREE TESTS OF GENERAL RELATIVITY

Our interpretation of the combinatorial hierarchy specifies the gravitational coupling as an *elementary particle* coupling $\hbar c/Gm_p^2 \simeq 1.7 \times 10^{38}$ analagous to $\hbar c/e^2 = 137$ rather than as a coupling to ponderable masses. Further, the content string corresponding to a massless quantum is simply the anti-null string $(1)_n$ corresponding to the forward light cone, or the null string $(0)_n$ corresponding to the backward light cone. This content string provides no information about the energy or momentum of the quantum. The way we meet this problem is similar to the *context dependent* approach of Wheeler and Feynman. We do not think of a quantum as "something traveling" but rather as a conceptual carrier of momentum and angular momentum between two earlier and later sets of particle motions. As Wheeler and Feynman would put it, all radiation is ultimately absorbed; radiation can only be discussed in terms of the sources and sinks ("action at a distance").

Once this is understood, we realize that the effect of gravitation for the problems at hand is to be measured by the changes in 3-momentum of two distant objects. This means that we will always be measuring mass ratios, and the fact that our gravitational coupling comes to us in units of the proton mass is no barrier to extending it to other objects and the correlated motions of their internal particulate emitters and absorbers of gravitons. Because of the weakness of gravitational effects in the systems of interest in this paper, we can simply sum these effects.

Once this is understood we can start our treatment of a gravitational system containing two significant masses $m = nm_p, M = Nm_p, \mu = \frac{nN}{n+N}m_p$ by following our treatment of the Bohr atom with the basic replacement

$$(2^{127} + 136)N_G steps = 1 \ newtonian \ event$$

From this quantum equation we can deduce the binding energy of two "Newtonian mass points" when

$$nN << (2^{127} + 136)N_G$$

to the same accuracy as we can deduce the binding energy of two charged particles with $q = ze, Q = Ze$ when $zZ << 137N_B$. The deduction of the solar red shift is then as trivial as it usually is in theories that combine special relativity with quantum mechanics. $E = h\nu$ quantization — which we derived above — and the conservation laws give the usual result, once we recognize that the absorption of a photon emitted by the sun somewhere near the earth's orbit (neglecting the local gravitational effects due to the earthand solar radius compared to the distance from the sun to the earth) must take account of the difference in energy between the absorbing systems and the emitting systems at the surface of the sun.

The deflection of light by the sun is only slightly more subtle. Although the details are not yet worked out, macroscopic gravitational (or Rutherford) scattering will connect to our quantum theory in the same way (to the accuracy of interest here) that our treatment of the Bohr atom connects to the "correspondence limit".

Hence the "special relativistic trajectory" of light passing by the sun (i.e. the deflection due to the $m = E/c^2$ attraction of the photon) will predict only half the observed effect. In this case we do not have a massive system in the region where the "interaction" occurs, and, until a more general treatment of the detection systems is carried through, must rely on theory to calculate the "lowest order" interaction between a graviton and a photon. In contrast to the red shift, which can be calculated from energetic considerations independent of spin, the photon passing the sun will have one of two well defined helicities relative to its direction of motion. The "newtonian" graviton (which provides the $E = mc^2$ special relativistic deflection) ignores this but the two helical gravitons cannot. The correct helicity can flip the photon's spin, but the other cannot. Hence, whatever the helicity of the photon which passes the sun, including spin only doubles the special relativistic spin independent deflection, in agreement with experiment.

The precession of the perihelion of Mercury involves massive systems at both the emission and the absorption points, so we have to use more care in talking about the helicity state of the gravitons. As is well known[20] the Sommerfeld calculation (in our language, the "newtonian term".) predicts only one sixth of the observed effect. In contrast to the deflection of light, where we had to deal with a "field-field" (photon-graviton) interaction, we talk here about two macroscopic bodies with a well defined orbit in 3-space. Relative to this situation the many spin two gravitons which eventually produce the perihelion shift can have *five* orientations and not just the two helicity states referred to their own "direction of motion". Consequently, in addition to the "newtonian" term we will have five equally probable contributions, and we can anticipate six times the Sommerfeld precession, in agreement with observation.

Pursuing this line of thought, it is interesting to contrast the situation in hydrogen with what would happen if we had a *macroscopic* electromagnetic elliptical orbit (eg in a space station) and 3-dimensional *macroscopic* measurements of the perihelion shift. It would seem to follow that *three* orientations of the spin 1 photons should contribute, each with the same probability as the coulomb term, and

hence lead to a prediction four times larger that the semi-classical model prediction on which the Sommerfeld formula is based.

REFERENCES

1. **DISCRETE AND COMBINATORIAL PHYSICS**: *Proceedings of the 9^{th} Annual International Meeting of the Alternative Natural Philosophy Association*, H.P.Noyes, ed.; ANPA WEST, 25 Buena Vista Way, Mill Valley, CA 94941, and references therein.
2. H.P.Noyes and D.O.McGoveran, "An Essay on Discrete Foundations for Physics", *Physics Essays*, **2**, No.1 (1989).
3. D.O.McGoveran, H.P.Noyes and M.J.Manthey, "On the Computer Simulation of the EPR-Bohm Experiment", presented at *Bell's Theorem, Quantum Theory and Conceptions of the Universe*, George Mason Univ., Fairfax VA (Oct.'88), in press, and SLAC-PUB-4729.
4. C. Gefwert, "Prephysics", Ref. 1, pp 1-36; what Gefwert calls *practice* — roughly, the initial working agreement as to how to go about meeting problems in the area under investigation— McGoveran would call the *epistemological framework* (E-frame).
5. D. McGoveran, "Foundations for a Discrete Physics", Ref. 1, pp 37-104.
6. W.Feller, *An Introduction to Probability Theory and its Applications*, Vol.I, 3^{rd} Edition, p. 316; for more detail see the first edition, p. 247 et. seq, Wiley, New York, 1950.
7. T.Bastin, *Studia Philosophica Gandensia*, **4**, 77 (1966).
8. For one attempt to reconstruct the history, see H.P.Noyes, "DISCRETE PHYSICS: Practice, Representation and Rules of Correspondence", in Ref 1, pp 117-118.
9. T.Bastin, H.P.Noyes, J.Amson and C.W.Kilmister, "On the Physical Interpretation and Mathematical Structure of the Combinatorial Hierarchy", *International Journal of Theoretical Physics*, **18**, 455-488 (1979).
10. C.W.Kilmister, "The Mathematics needed for self-organization", pp. 11-17 of *Disequilibrium and Self Organization* (C.W.K. ed.) Reidel, Dordrecht, 1986; see also Appendix II.1 "Brouwerian Foundations for the Hierarchy" (based on a paper presented at ANPA 2, King's College, Cambridge, 1980) and Appendix II.2 "On Generation and Discrimination" (based on a paper presented at ANPA 4, King's College Cambridge, 1982) in H.P.Noyes, C.Gefwert and M.J.Manthey, "Toward a Constructive Physics", SLAC-PUB-3116 (rev. Sept. 1983).

11. C.W.Kilmister, "A Final Foundational Discussion?", in *Proc. ANPA 7* and as an appendix in H.P.Noyes, "ON THE CONSTRUCTION OF RELATIVISTIC QUANTUM MECHANICS: A Progress Report", SLAC-PUB-4008 (June 1986).

12. A.F.Parker-Rhodes, *The Theory of Indistinguishables*, Synthese Library, **150**, Reidel, Dordrecht, 1981.

13. H.Doughty's "Double Fields", invented in 1968, give the same first sequence. If they also terminate (or do not) the corresponding sequence of Mersenne primes ends (is unbounded). Either result would answer an unsolved query in number theory. A paper on Double Fields will be submitted by Doughty to *Proc. ANPA 10.*

14. M.J.Manthey, "Program Universe" in *Proc.ANPA 7*, Dr. F.Abdullah, Room E517, The City University, Northampton Square, London EC1V 0HB.

15. D.O.McGoveran and H.P.Noyes, "On The Fine Structure Spectrum of Hydrogen", SLAC-PUB-4730, submitted to *Physical Review Letters* Nov. 7, 1988.

16. N.Bohr, *Phil.Mag*, Feb. 1915.

17. A. Sommerfeld, *Ann. d. Phys.IV*, **17**, 1-94 (1916).

18. P.A.M.Dirac, *Principles of Quantum Mechanics*, Oxford, 3^{rd} edition, 1947, pp 268-271.

19. M.Aguilar-Benitz, et.al., "Review of Particle Properties", Physics Letters **170 B**, April 1986.

20. P.G.Bergmann, *Introduction to the Theory of Relativity*, Prentice Hall, New York, 1942, pp 217-218.